§3- From Abstract To Concrete Knot Theory

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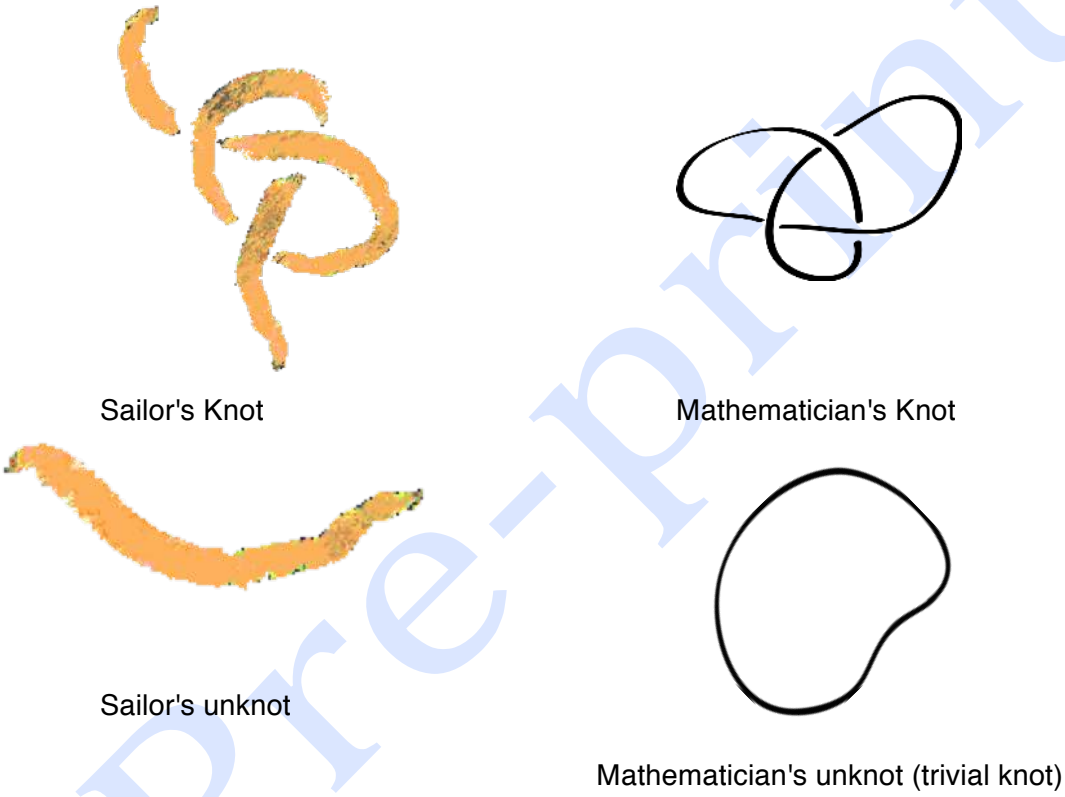
Unlike ordinary knots, the mathematician's knot must already be tied and must not come untied by simply undoing its ends. For this reason, the knot for the mathematician is not an open ended string, but a closed curve:



Concrete and Abstract Knots
Fig.1

What produces the tying and untying of a knot is not accounted for by a theory of formal knots – it only acknowledges the massive existence of the knot from the beginning in the shape of a closed curve.

Following a standard philosophical protocol, the problem of setting up

an formal and mathematical theory of knots apparently requires an *abstraction* that excludes the *concrete*: the diagrams, models, inscriptions, reference, etc.

Thus, we refer again to the standard position of R. Fox and Crowell as examples of this tradition:

"Mathematics never proves anything about anything except mathematics, and a piece of rope is a physical object and not a mathematical one. So before worrying about proofs, we must have a mathematical definition of what a knot is and another mathematical definition of when two knots are to be considered the same. The problem of formulating a mathematical model arises whenever one applies mathematics to a physical situation. The definition should define mathematical objects that approximate the physical objects under consideration as closely as possible. The model may be good or bad according as the correspondence between mathematics and reality is good or bad. There is, however, no way to prove (in the mathematical sense, and it is probably only in this sense the word has a precise meaning) that the mathematical definitions describe the physical situation exactly".

Richard H. Crowell & Ralph H. Fox, Introduction to Knot Theory

Indeed, following this standard parameter, not only must definitions- and mathematical models be left approximate since they refer to concrete physical situations, but the use of sketches and knot projections are viewed as a hindrance to V. Jones:

Although the formalism (for mechanical statistical models or "vertex models") is quite general and not tied to braid presentations or induction, it is still hampered by the need for a two-dimensional projection (shadow) of a three-dimensional object. Our main reason for doing this work was as a step towards a useful and genuine understanding of three-dimensional invariants. So far we have not succeeded. The situation is the same as the poor prisoners in Plato's allegory of the cave.

V. Jones*, On Knot Invariants Related To Some Related Statistical Mechanical Models*

Despite their sanction from the side of usage and common sense, the attempt to exclude concrete presentations from an abstract and formal theory leads to trouble. For we will show that applied to certain

problems of knot theory, most notably the problem of determining the identity and existence of the knot, the notion of an abstract knot that excludes its presentations – diagrams, models, tokens, language, etc. – produces results that can not be consistently maintained.

By opposing the *concrete* to the *abstract* as *everyday life* is opposed to *mathematics*, just as the *informal* is opposed to the *formal*, or *presentation* to *structure*, the concrete element of a theory becomes a source of error, approximation, or mere intuition. In adopting the position of analytic and structural theory, we must begin otherwise.

First, what is *invariant* of a theory – its *structure* – only emerges through a *presentation* and not as the result of an abstraction. This presentation may occur through the use of *whatever* – Hilbert's beer mugs, tables, and chairs – but not *however.* This *'not however'* constitutes a *parameter* – the modes of choice and collection of traits by which no matter what can be presented. Yet, this alterity of a structure should not be mistaken with the approximations of a model to a domain (reality, a physical situation, etc.), since a structure is relative to the language of a theory and not its domain. This difference is important for two reasons: first, unlike a model that is always true relative to a domain, a structure may be true or false since it is only relative to the language of a theory (it is not only permitted, but necessary to write false statements and construct false diagrams without asserting them in a theory). Second, the alterity of structure emerges across domains and models – think Stravinsky not Mozart – since it is what is invariant with regard to the discourse of a theory that is determinant.

No doubt, the main theoretical difficulty inherent in introducing a structural theory is the delimitation of this *whatever*: those informal and concrete features that an abstract and formal theory choose to ignore or exclude or regulate to the field of culture or interdisciplinary research. In a manner that is more acute, it appears to me that the philosophical positions maintained by Crowell, Fox, and Jones above have forgotten the more contemporary lessons of the modern mathematicians-logicians (Pierce, Frege, Quine, Weyl, Hilbert, Klein, and Tarksi among others) and artists (Kandinsky, Klee, Miro, Leger,

and Arp among others) who adopt a structural position and proceed by an inversion of the opposition concrete/abstract.

Artists:

This 'world of art' is just as real, just as concrete. For this reason I prefer to call so called 'abstract' art 'concrete' art.

Kandinsky

In 1921 I visited Kandinsky in Munich. He gave me a very warm reception. It was the period when abstract art was beginning to turn into concrete art; that is to say, the avant-garde painters no longer stood before an apple, a guitar, a man, or a landscape to convert or dissolve them into colored circles, triangles, and rectangles; on the contrary, they created autonomous compositions directly out of their most intimate joy, their most personal suffering, out of lines, planes, forms, colors.

Arp

Apparent abstractions "are not 'abstract', since they are composed of real values: colors and geometric forms. There is no abstraction".

F. Leger

Logicians

As to that process of abstraction, it is itself a sort of observation. The faculty which I call abstractive observation is one which ordinary people perfectly recognize, but for which theories of philosophers hardly leave room. It is a familiar experience to every human being to wish for something quite beyond his present means, and to follow that wish by the question, "Should I wish for that thing just the same, If I had ample means to gratify it?" To answer that question, he searches his heart, and in doing so makes what I term an abstractive observation. He makes in his imagination a sort of skeleton diagram, or outline sketch, of himself, considers what modifications the hypothetical state of things would require to be made in that picture, and then examines it, that is, observes what he has imagined, to see whether the same ardent desire is there to be discerned. By such a process, which is at the bottom very much like mathematical reasoning, we can reach conclusions as to what would be true of signs in all cases, so long as the intelligence using them was scientific.

C. Pierce

For suppose that we do, as THOMAE demands, "abstract from the peculiarities of the individual members of a set of items", or "disregard in considering separate things, those characertistics which serve to distinguish them". In that event we are not left, as LIPSCHITZ maintains, with "the concept of the Number of the things considered"; what we get is rather a general concept under which the things in question fall. The things themselves do not in the process lose any of their special characteristics. If, for example, example in considering a white cat or black cat, I disregard the properties which serve to distinguish them, then I get presumably the concept "cat". Even if I proceed to bring them both under this concept and call them, I suppose units, the white one still remains white just the same, and the black black. I may not think about their colours, or I may propose to make no inference from their difference in this respect, but for all that the cats do not become colorless and they remain different precisely as before.

G. Frege

To construct a concrete and primitive knot theory would not be to construct the knot theory of primitives or a scientific vulgarization but to introduce a *structural* theory. Instead of asking what are the invisible abstractions of *form* behind the visible concrete *figures* projected onto the walls of Plato's cave – and taking this recognition of form as the most universal attribute not only of mathematics, but of the human mind – we ask what is a *trait*? We then look for the response to the side of the slaves by asking not what do they see, but what drew them to look at the wall to begin with? the search for knowledge or the whip? What assures the slaves that the designs that they are looking at refer to the same thing as the master? Form or hypnosis?

Or let us change caves altogether and ask the primitives of LasCaux, not how did they get out of the cave to see the truly invisible forms, but by what concrete light did they get into the cave to begin with? That is to say, by what opening and use of light were they able to draw their designs? And if the archaeologist have taught us that their drawings were not mere *representations* of the reality outside the cave, but *presentations* of the rituals, rhythms, and incantations that were performed inside, what are the ramifications of such a concrete

and performative position for the construction of a knot theory?

No doubt, this is not the place to respond to such questions, we only outline the folklore here in order not to assimilate the problem of *structure* to that of *form*. The challenge that is before us is to show how *On Knots*, far from being an immature and primitive theory in the tradition of science and formal knot theory, is a precursor to a concrete and structural knot theory; or more precisely, of a knot structure that is present in any mathematical knot theory – past or present – in so far as it has not been domesticated and codified into an abstraction.

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